

Statistical Methods in AI (CSE/ECE 471)

Lecture-3: Intro to Performance Measures,
Benchmarking



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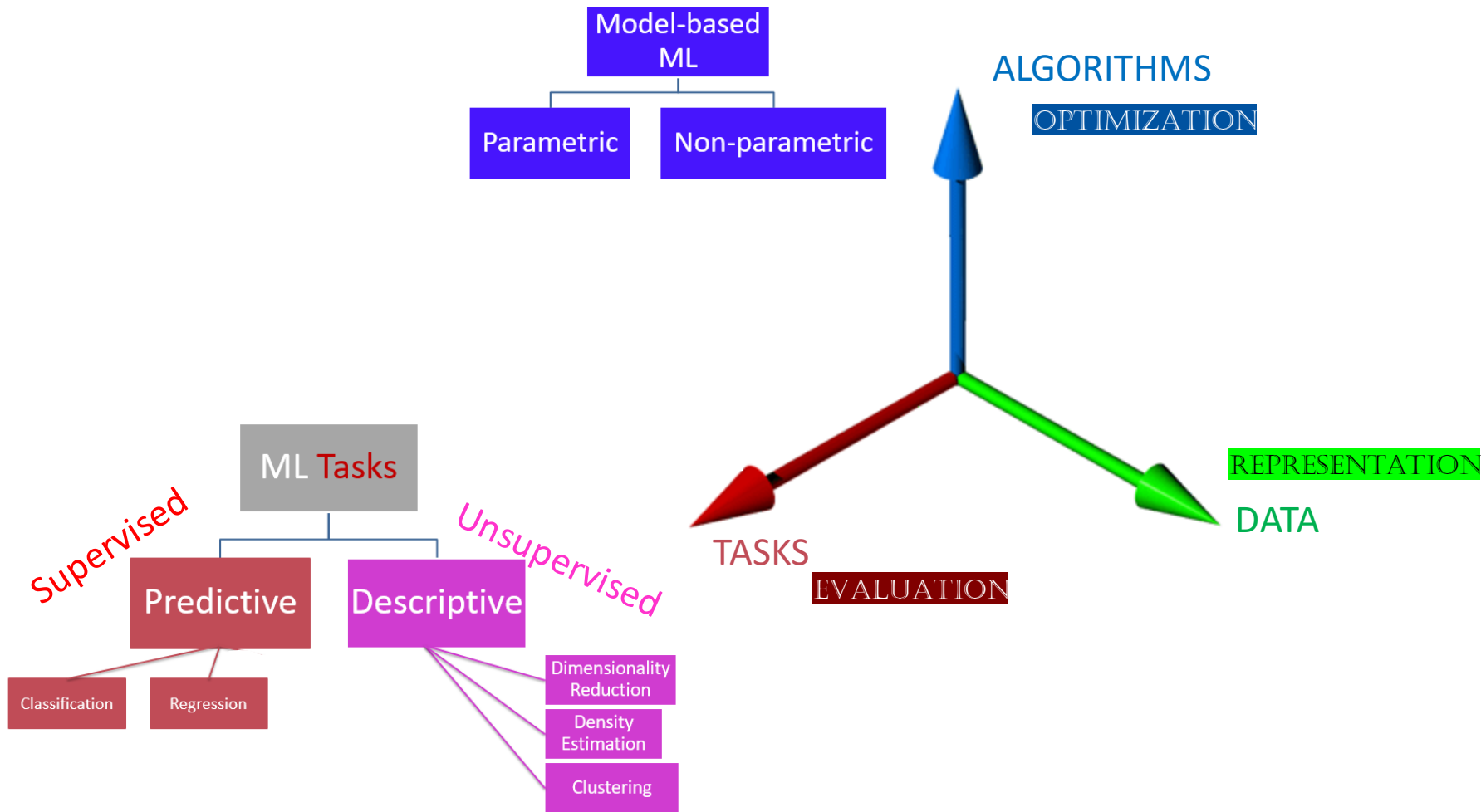
Center for Visual Information Technology (CVIT)
IIIT Hyderabad



Machine Learning



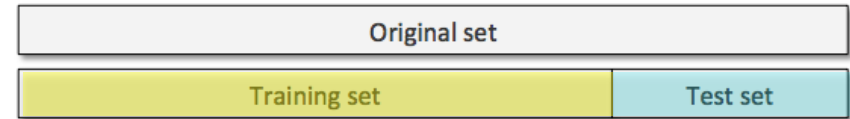
Study of **Algorithmic methods** that use **data** to **improve** their **knowledge** of a **task**



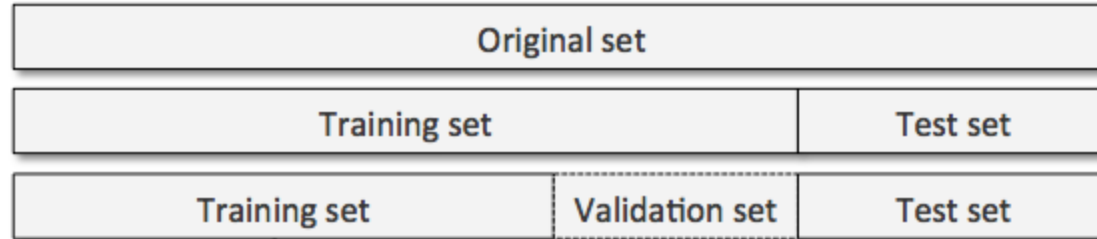
An interview analogy

1. Collect worked out problems (Q, S are both known)
2. Prepare on ALL the available problems.
3. Go for interview.

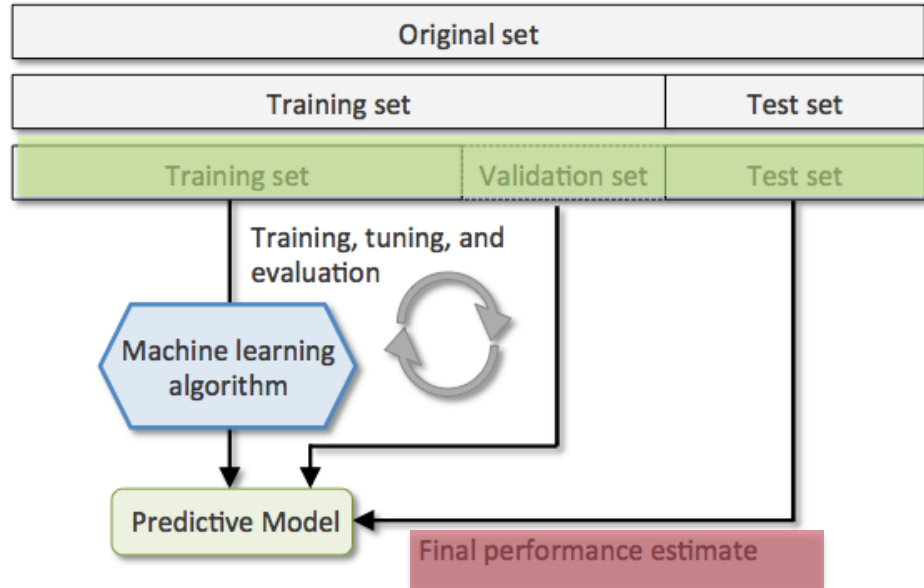
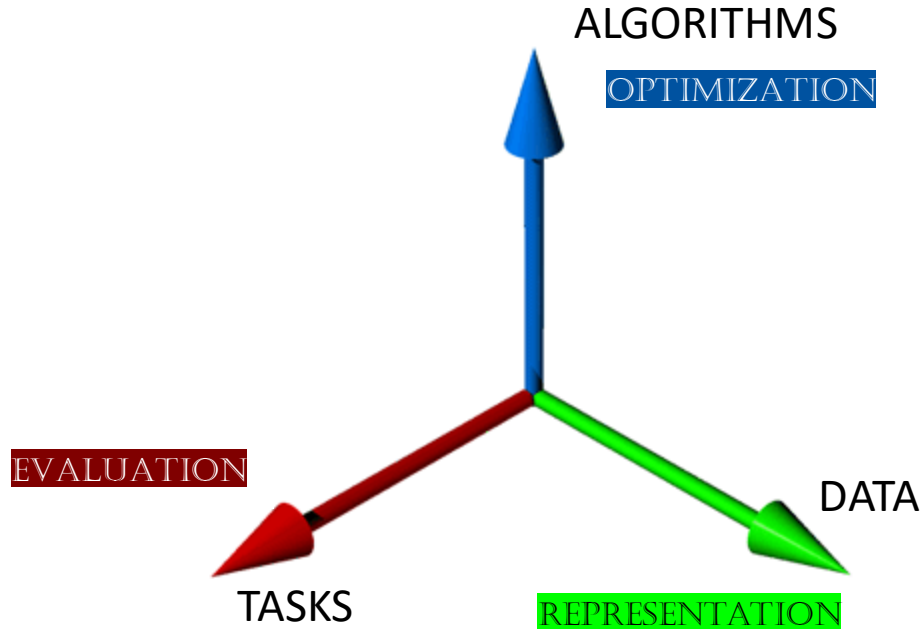
1. Collect **worked out problems** (Q,S are both known)
2. Randomly set aside a small number of problems.
3. Prepare on rest of the problems.
4. Take a mock interview containing all the **'set aside' problems**.
5. Score answers and compare with solution.
6. Use mistakes to decide which topics to prepare better on.
7. Go for interview.



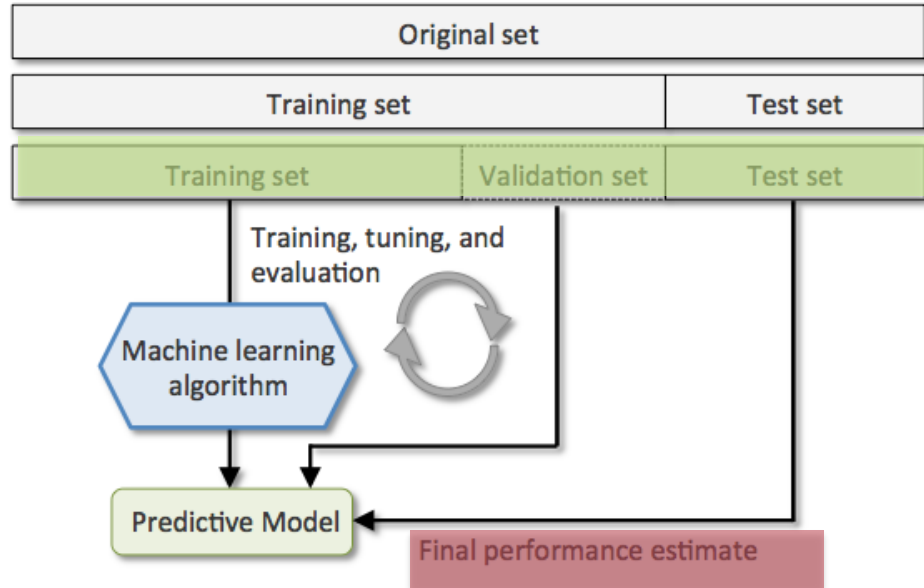
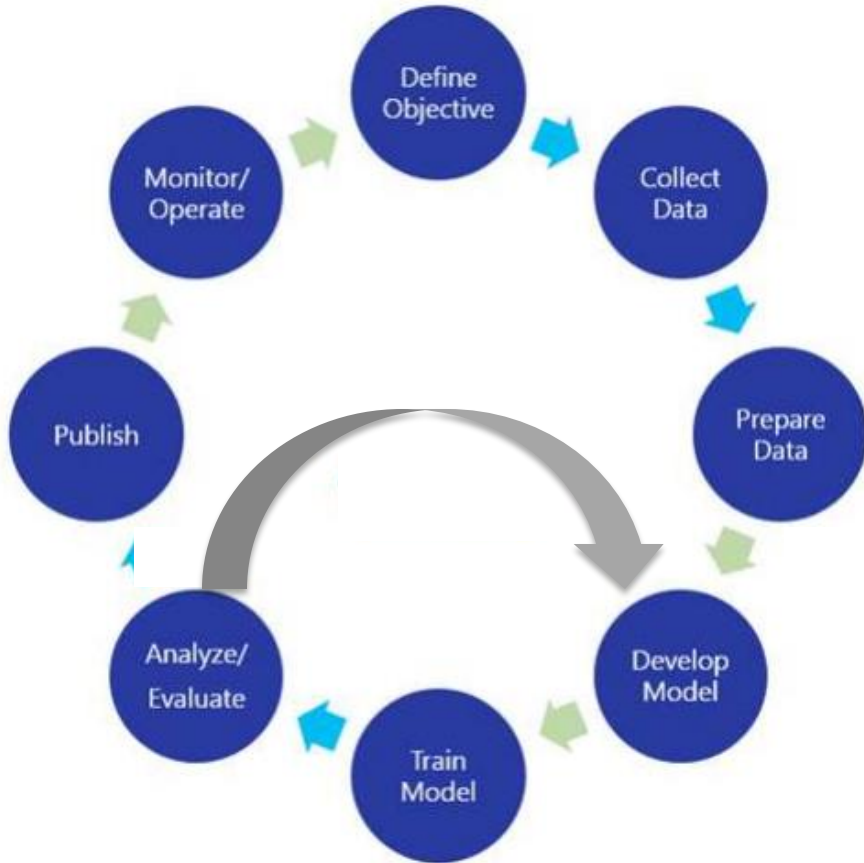
The Train-Validation-Test paradigm

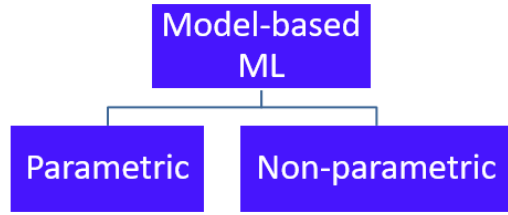


The Train-Validation-Test paradigm

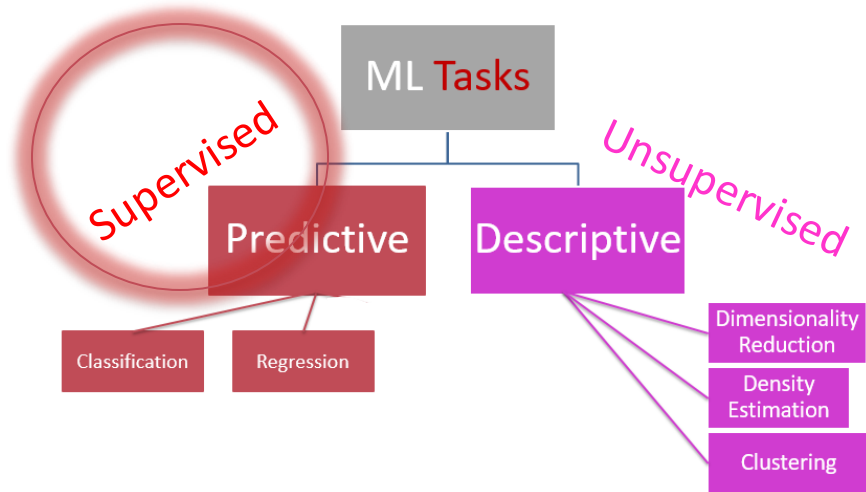
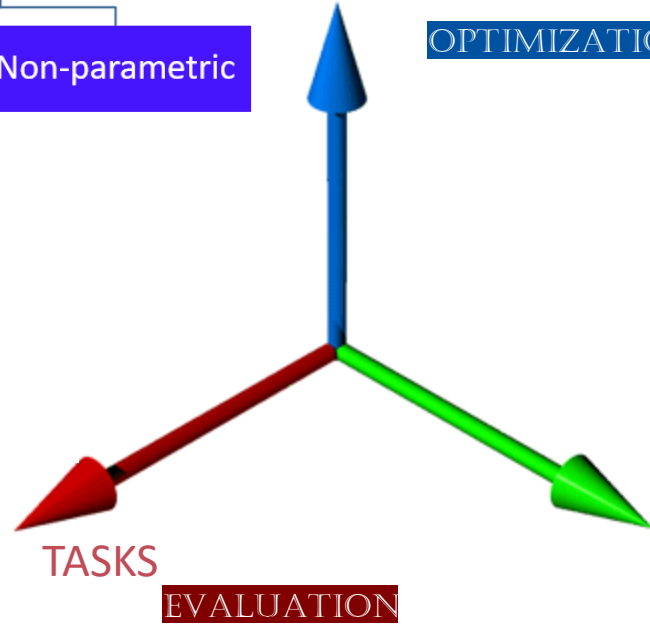


The Train-Validation-Test paradigm





ALGORITHMS
OPTIMIZATION



Supervised Learning

```
graph TD; A[Supervised Learning] --> B[Classification]; A --> C[Regression]; style B stroke-dasharray: 5 5;
```

Classification

Regression

ML::Tasks \rightarrow Predictive \rightarrow Classification

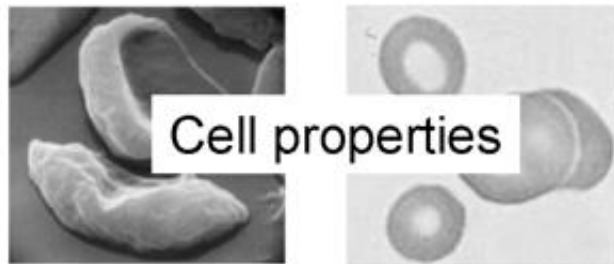
Feature Space \mathcal{X}



Label Space \mathcal{Y}



"Sports"
"News"
"Science"
...

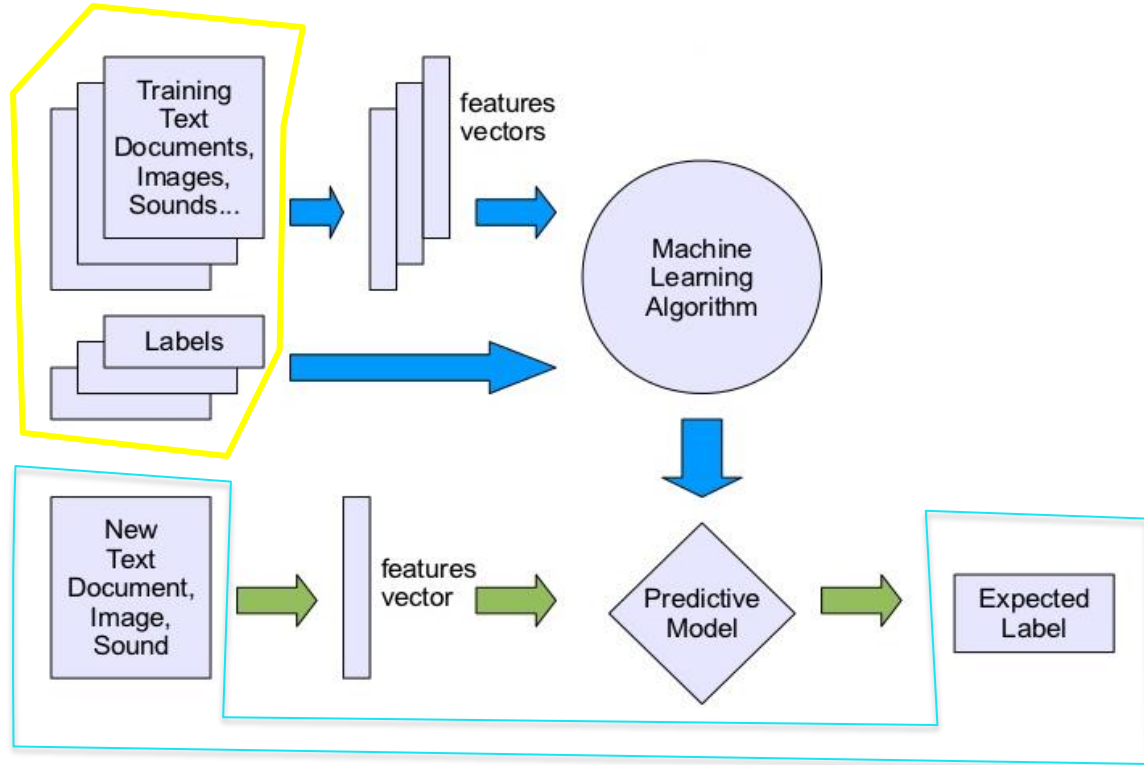
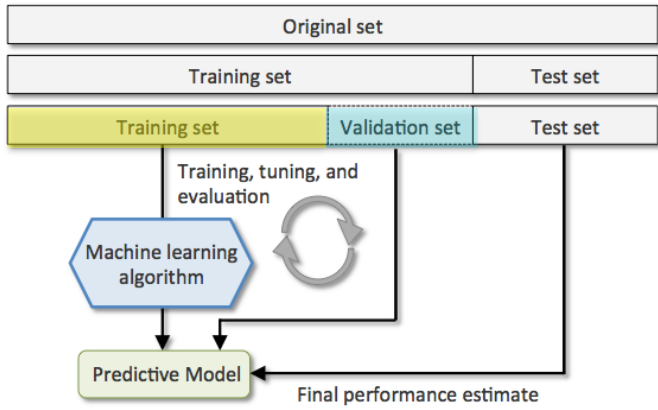


"Anemic cell"
"Healthy cell"

Task: Given $X \in \mathcal{X}$, predict $Y \in \mathcal{Y}$.

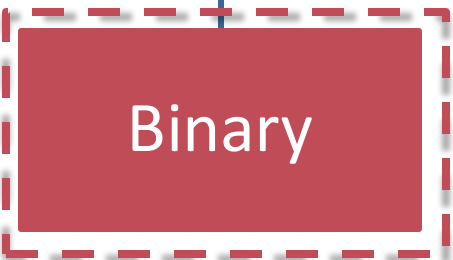
Discrete Labels

The Train-Validation-Test paradigm





Classification



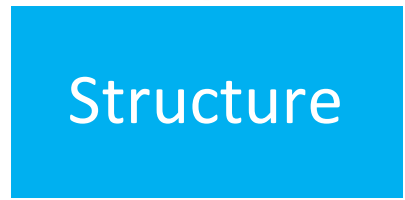
$\{0,1\}$



1-of-K



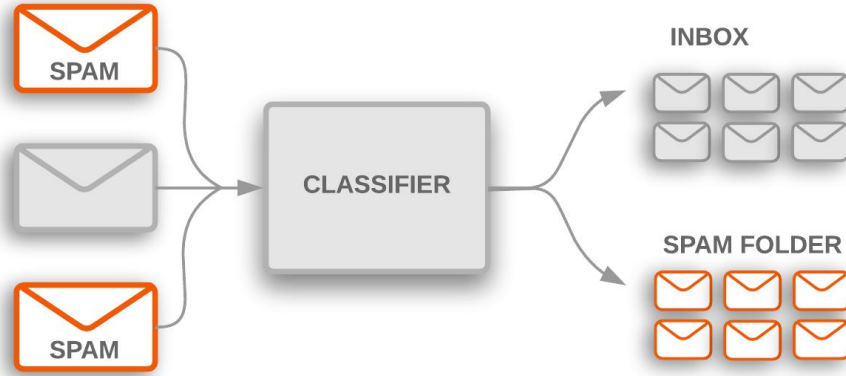
n-of-K



E.g. graph/sequence



Binary Classification



Performance Measures - Accuracy

$$Accuracy = \frac{(100 + 50)}{165} = 0.91$$

$$Misclassification = \frac{(10 + 5)}{165} = 0.09$$

| n=165 | Predicted: NO | Predicted: YES |
|----------------|------------------|-------------------|
| Actual: NO | 50 | 10 |
| Actual: YES | 5 | 100 |

- **Pool of 100 patients' data used for validation of a cancer prediction ML model**

- Prediction:

- 3 have cancer
- Rest (100-3=97) are healthy.

- Reality:

- 1 of the 3 did not actually have cancer !
- 3 from 97 predicted healthy actually have cancer

- Accuracy =

| n= ___ | Predicted: NO | Predicted: YES |
|----------------|------------------|-------------------|
| Actual: NO | | |
| Actual: YES | | |

- **Pool of 100 patients' data used for validation of a cancer prediction ML model**
- Prediction:
 - 3 have cancer
 - Rest (100-3=97) are healthy.
- Reality:
 - 1 of the 3 did not actually have cancer !
 - 3 from 97 predicted healthy actually have cancer
- Accuracy = $(100 - 4) / 100 = 96\%$!

| n= ___ | Predicted: NO | Predicted: YES |
|----------------|------------------|-------------------|
| Actual: NO | | |
| Actual: YES | | |

- **Pool of 100 patients' data used for validation of a cancer prediction ML model**

- Prediction:
 - 3 have cancer → selected for chemotherapy
 - Rest (100-3=97) are healthy.
- Reality:
 - 1 of the 3 did not actually have cancer !
 - 3 from 97 predicted healthy actually have cancer → should have been selected for chemotherapy

| n= ___ | Predicted: NO | Predicted: YES |
|----------------|------------------|-------------------|
| Actual: NO | | |
| Actual: YES | | |

Performance Measures - Accuracy

$$Accuracy = \frac{(100 + 50)}{165} = 0.91$$

$$Misclassification = \frac{(10 + 5)}{165} = 0.09$$

| n=165 | Predicted: NO | Predicted: YES |
|----------------|------------------|-------------------|
| Actual: NO | 50 | 10 |
| Actual: YES | 5 | 100 |

Performance Measures – Accuracy, TPR, FPR

$$Accuracy = \frac{(100 + 50)}{165} = 0.91$$

$$Misclassification = \frac{(10 + 5)}{165} = 0.09$$

$$FalsePositiveRate(FP) = \frac{(10)}{60} = 0.17$$

$$FalseNegativeRate(FN) = \frac{(5)}{105} = 0.048$$

| | | Predicted: | | |
|---------|-----|------------|----------|-----|
| | | NO | YES | |
| Actual: | NO | TN = 50 | FP = 10 | 60 |
| | YES | FN = 5 | TP = 100 | 105 |
| | | 55 | 110 | |

$$\text{TrueNegativeRate}(TN) = \frac{(50)}{60} = 0.833$$

$$\text{TruePositiveRate}(TP) = \frac{(100)}{105} = 0.95$$

| | | Predicted: | | |
|---------|-------|------------|----------|-----|
| | n=165 | NO | YES | |
| Actual: | | | | |
| NO | | TN = 50 | FP = 10 | 60 |
| YES | | FN = 5 | TP = 100 | 105 |
| | | 55 | 110 | |

Type I error
(false positive)



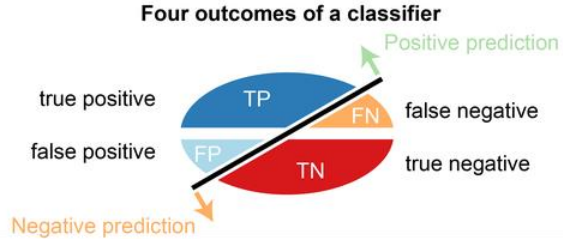
Type II error
(false negative)



Figure 3.1 Type I and Type II errors

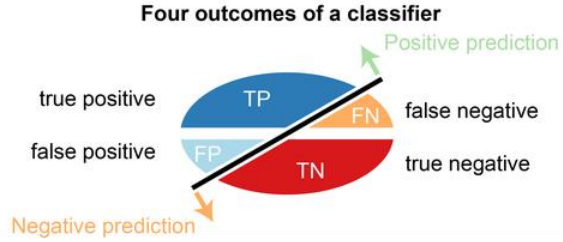
levels to .01 or even .001

Summary of Measures

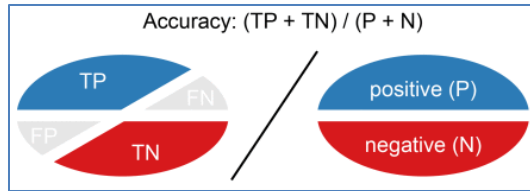


| | | Predicted: NO | Predicted: YES | |
|-------------|--|---------------|----------------|-----|
| n=165 | | | | |
| Actual: NO | | TN = 50 | FP = 10 | 60 |
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Summary of Measures

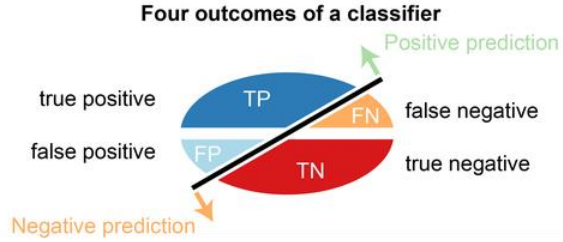


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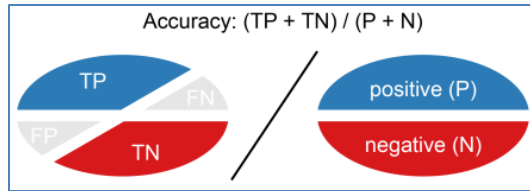


% of correct predictions

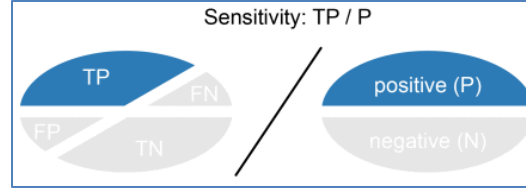
Summary of Measures



| | | Predicted: NO | Predicted: YES | |
|-------------|--|---------------|----------------|-----|
| n=165 | | | | |
| Actual: NO | | TN = 50 | FP = 10 | 60 |
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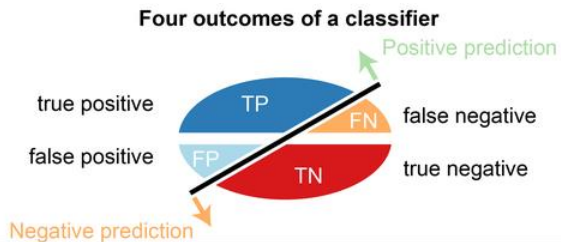


% of correct predictions

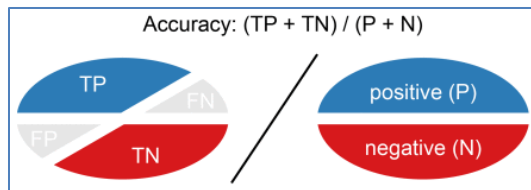


% of + class correctly predicted
[aka Recall / TPR]

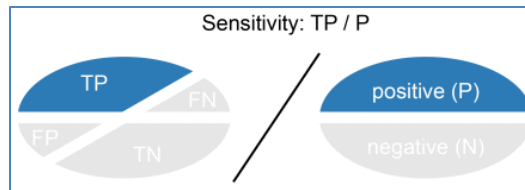
Summary of Measures



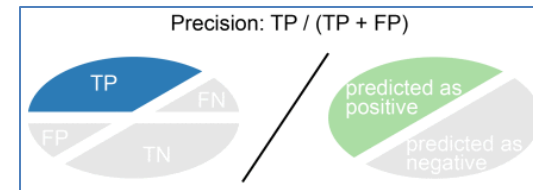
| | Predicted: NO | Predicted: YES | |
|-------------|---------------|----------------|-----|
| n=165 | | | |
| Actual: NO | TN = 50 | FP = 10 | 60 |
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% of correct predictions

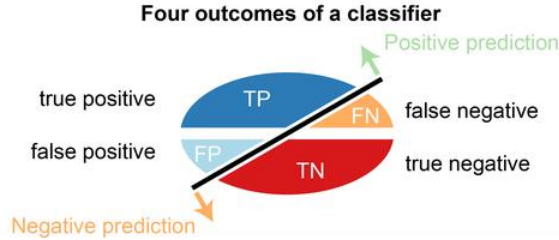


% of + class correctly predicted
[aka Recall / TPR]

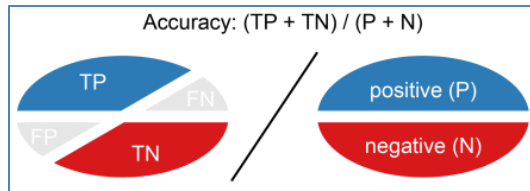


correct prediction of + class
[aka Precision]

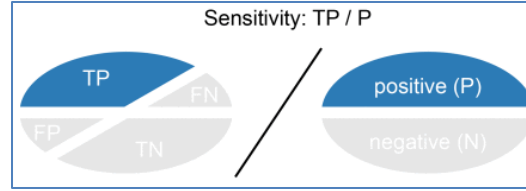
Summary of Measures



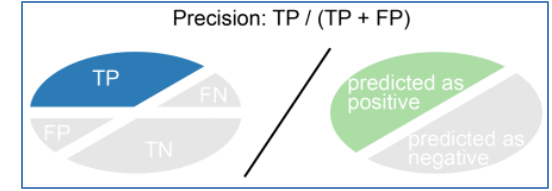
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% of correct predictions



% of + class correctly predicted
[aka Recall / TPR]



correct prediction of + class
[aka Precision]



% of - class incorrectly predicted

- **Cancer-Prediction System**

- Precision =

- Recall =

- Accuracy =

- **Cancer-Prediction System**
- Precision = $2/(2+1) = 67\%$
- Recall = $2/(2+3) = 40\%$
- Accuracy = $(94+2)/100 = 96\%$

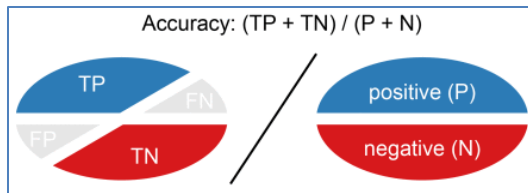
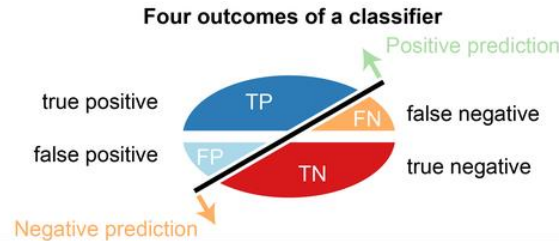
Precision and Recall – examples

- A system which needs to launch a missile at a terrorist hideout located in a dense urban area.
- Precision not 100% → civilian casualties

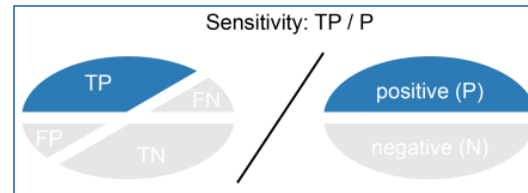
- A system which needs to identify cancer-risk patients
- Recall not 100% → some patients will die of cancer

Accuracy vs Precision vs Recall

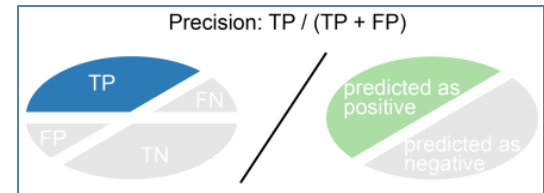
- Accuracy : Performance w.r.t both classes
- Recall : Performance w.r.t '+' class
- Precision : Reliability of predictions w.r.t '+' class



% of correct predictions



% of + class correctly predicted
[aka Recall / TPR]



correct prediction of + class
[aka Precision]

Utility and Cost

- Sometimes, there is a cost for each error
 - E.g. Earthquake prediction
 - False positive: Cost of preventive measures
 - False negative: Cost of recovery
- Detection Cost (Event detection)
 - $\text{Cost} = C_{\text{FP}} * \text{FP} + C_{\text{FN}} * \text{FN}$

Farmer Shri MoneyBags and ML-FruitPicker

- MB : I want an automated fruit picker and packer. I will pay an unholy amount for it.
- You (having just finished this lecture) : Sure
- *You (Thinking): I love unholy amounts of money 😎*
- *(rapid cuts of time passing, you collecting data, referring to SMAI slides, coding ; dramatic music in background)*

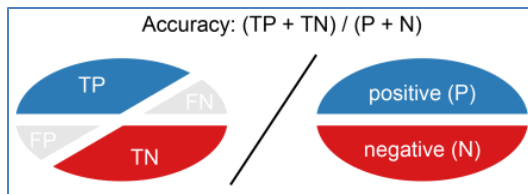
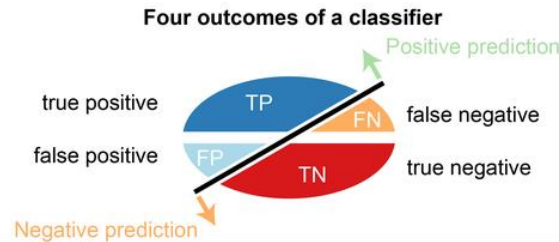
Farmer Shri MoneyBags and ML-FruitPicker

After 6 months ...

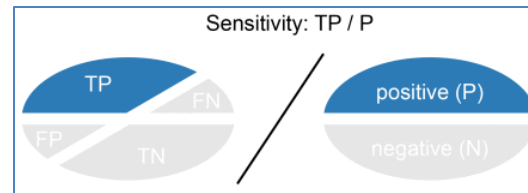
- MB : Well ?
- You : I have a High Precision ML-FruitPicker. But its Recall is 20% !
- MB : (confused) Precision ? Recall ?
- *You : (thinking) Should I go over first 3 lectures of SMAI with MB ? He'll probably run away !*
- You : It rejects 80% of good, pickable fruit, but whatever it picks, those fruits are good !
- MB : I'll take your system. How do I transfer unholy amount of money to you ?
- You : 😳
- MB (seeing your shocked face) : See, in a batch of 100 fruits, 10 fruits are usually bad. Among the 90 good ones, your system will select 18 of them on average. But from any given selection, I pack only 8.

Accuracy vs Precision vs Recall

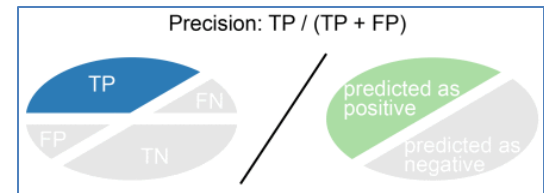
- Monitor **Precision** if a **false positive** carries higher cost.
- Monitor **Recall** if a **false negative** carries higher cost.



% of correct predictions



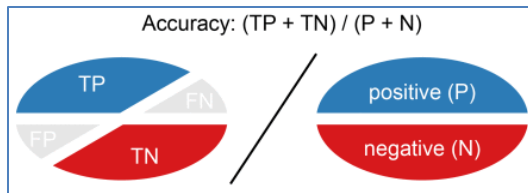
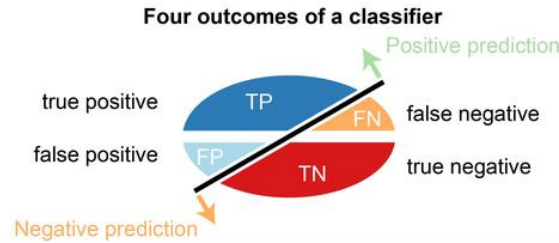
% of + class correctly predicted
[aka Recall / TPR]



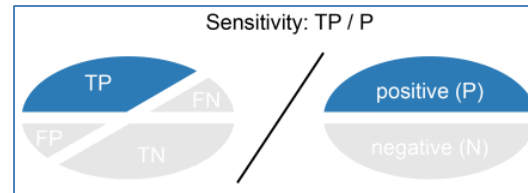
correct prediction of + class
[aka Precision]

Accuracy vs Precision vs Recall

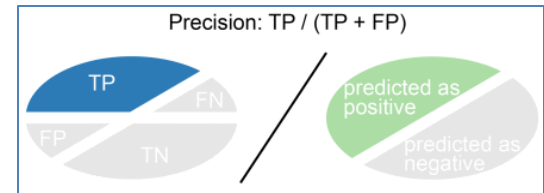
- **Precision** → Cost of inclusion
- **Recall** → Cost of exclusion



% of correct predictions

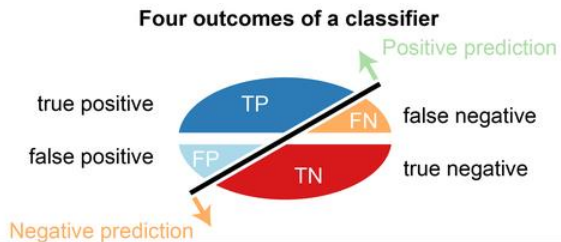


% of + class correctly predicted
[aka Recall / TPR]

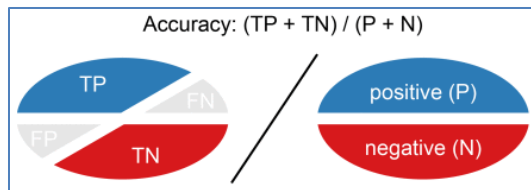


correct prediction of + class
[aka Precision]

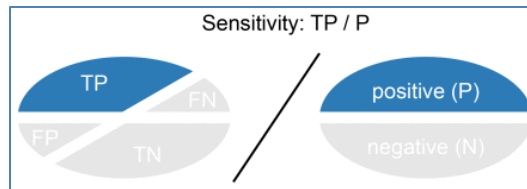
Summary of Measures



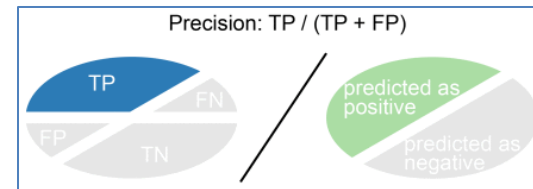
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|-------------|---------------|----------------|-----|
| n=165 | | | |
| Actual: NO | TN = 50 | FP = 10 | 60 |
| Actual: YES | FN = 5 | TP = 100 | 105 |
| | 55 | 110 | |



% of correct predictions



% of + class correctly predicted
[aka Recall / TPR]



correct prediction of + class



% of - class incorrectly predicted

F1-score: A unified measure

- What to do when one classifier has better precision but worse Recall, while other classifier behaves exactly opposite?
 - F-measure (Information Retrieval)

- $F_1 = \frac{1}{\frac{1}{Recall} + \frac{1}{Precision}}$

Utility and Cost

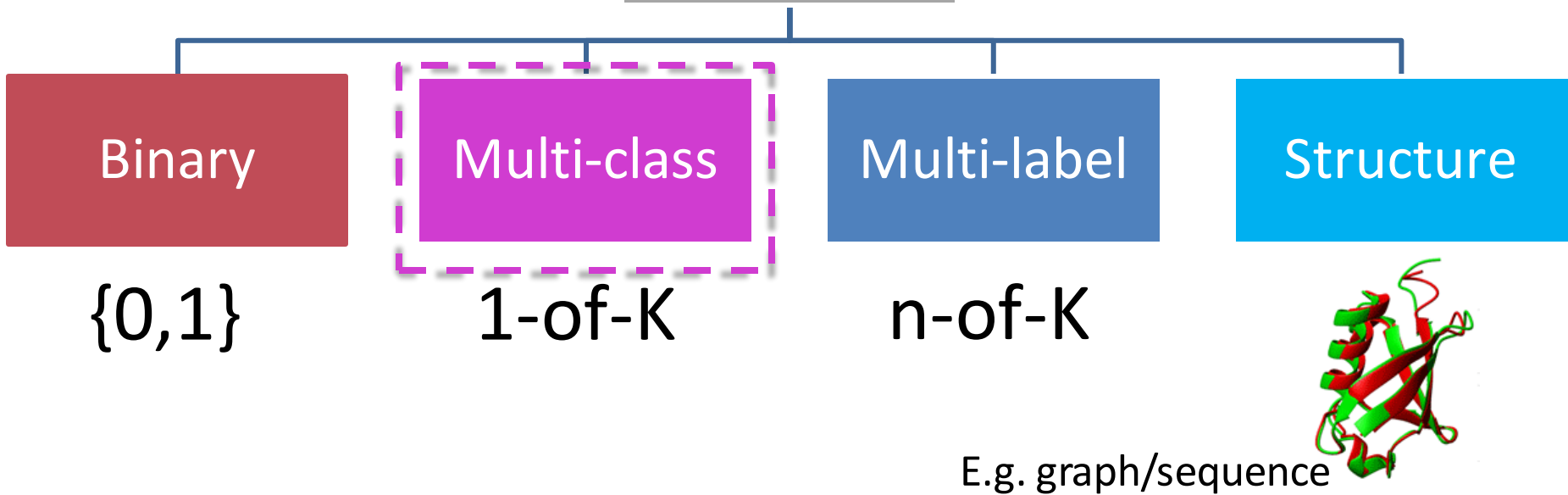
- What to do when one classifier has better Precision but worse Recall, while other classifier behaves exactly opposite?
 - F-measure (Information Retrieval)

$$\blacksquare F_1 = \frac{2}{\frac{1}{\text{Recall}} + \frac{1}{\text{Precision}}}$$

- **F1 measure punishes extreme values more !**
- **Definition of Recall and Precision have same numerator, different denominators. A sensible way to combine them is harmonic mean.**

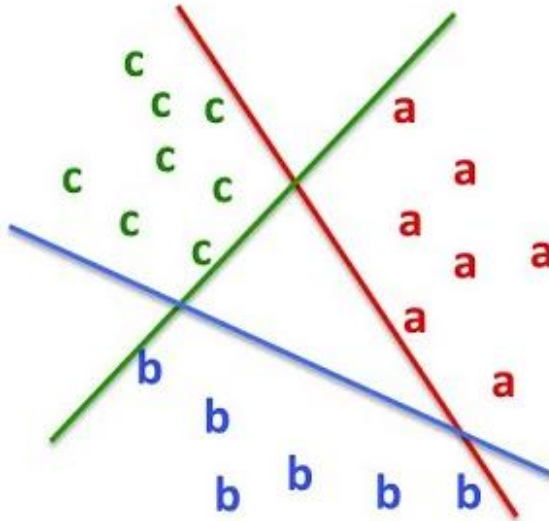


Classification



How to use 2-class measures for multi-class ?

- Convert into 2-class problems !

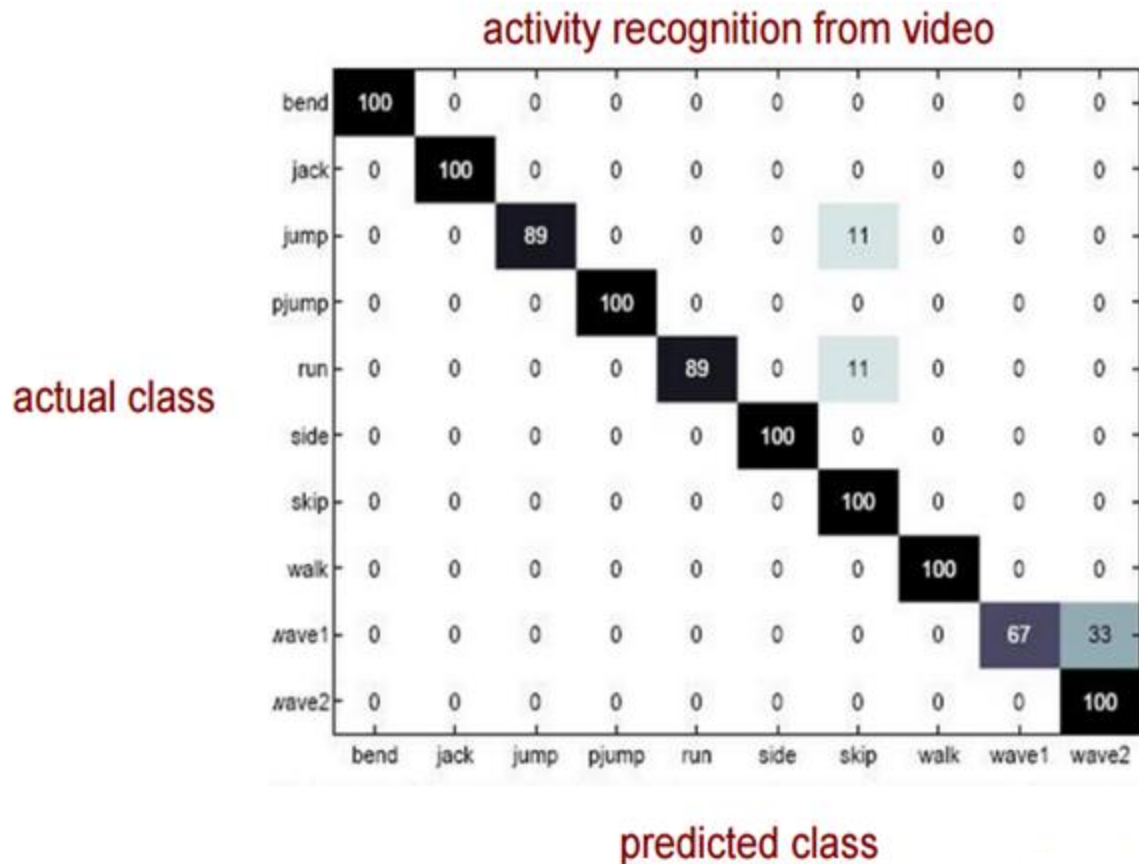


- Average Precision, Recall etc.



Avg. accuracy may not be very meaningful with imbalanced class label distribution

Multi-class problems - Confusion matrix

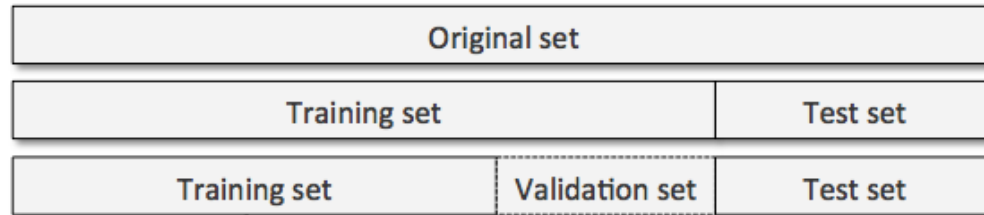


Multi-class Classification: Measures

- Mean <measure> +- standard deviation
- Median <measure> +- median absolute deviation

| Descriptor | Spectral bands | |
|----------------|------------------|------------------|
| | RGB | PCA RGB |
| Gist | 74.14 ± 1.93 | 77.76 ± 2.62 |
| MSIFT | 88.92 ± 1.39 | 90.97 ± 1.81 |
| MBoW | 88.60 ± 1.70 | 88.31 ± 1.38 |
| cSIFT | 88.17 ± 1.17 | 88.76 ± 1.74 |
| rgSIFT | 88.24 ± 1.89 | 87.71 ± 1.33 |
| BoWV [8] | 71.86 | N/A |
| SPMK [12] | 74.00 | N/A |
| SPCK++ [8] | 76.05 | N/A |
| Dense SIFT [2] | 81.67 ± 1.23 | N/A |

Exam analogy: Did you prepare at least a little ?



- Compute <Performance Measure> (e.g. Accuracy) for TRAINING SET
- Verify it is “decent”



Classification

Binary

$\{0,1\}$

Multi-class

1-of-K

Multi-label

n-of-K

Structure



E.g. graph/sequence

Example-based

- n is the number of examples.
- Y_i is the ground truth label assignment of the i^{th} example..
- x_i is the i^{th} example.
- $h(x_i)$ is the predicted labels for the i^{th} example.

$$\text{Precision} = \frac{1}{n} \sum_{i=1}^n \frac{|Y_i \cap h(x_i)|}{|h(x_i)|}$$

What fraction of labels are predicted correctly ?

$$\text{Recall} = \frac{1}{n} \sum_{i=1}^n \frac{|Y_i \cap h(x_i)|}{|Y_i|}$$

What % of correct labels were predicted ?

Accuracy = Fraction of samples predicted correctly

Baselines

- 0 cost-to-build classifiers
- Binary
 - Equal # of samples / class → Random Guessing (50% accuracy)
 - Class imbalance
 - → Guess according to class proportion (Accuracy =)
 - 0-Rule: Majority class (Accuracy =) [slightly stronger baseline]

Summary

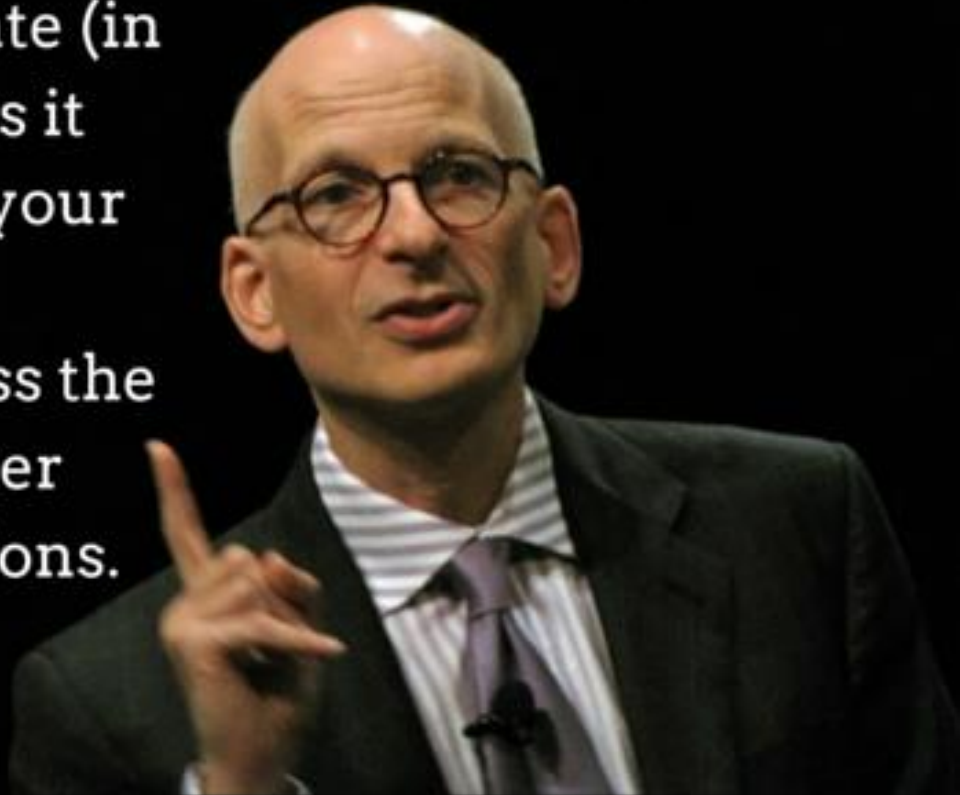
- Many metrics:
 - Accuracy, TP, FP, Precision, Recall, AP/mAP
 - Class imbalance and decision-cost imbalance must be taken into account
- Confusion Matrix: Important to analyze and refine solution.



A useful metric is both accurate (in that it measures what it says it measures) and aligned with your goals.

Don't measure anything unless the data helps you make a better decision or change your actions.

~ Seth Godin



References and Reading

- Code
 - https://scikit-learn.org/stable/modules/model_evaluation.html#classification-metrics